

Discrete Mathematics

[Time: $2\frac{1}{2}$ Hrs.]

[Marks:75]

Please check whether you have got the right question paper.

- N.B: 1. All question is compulsory.
2. Figures to the right indicate full marks.

Q.1 Attempt **any four** of the following: **20**

- a. Show that, $f: \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = 2x + 5$ is a bijection and hence its inverse.
- b. Prove that composition of bijective function is bijective.
- c. Check whether the binary operation defined on corresponding set is commutative or associative $a * b = a + b - 3$ for every $a, b \in \mathbb{Z}$.
- d. If R is relation on \mathbb{Z} defined as $(x, y) \in R$ if $3x + 4y$ is divisible by 7. Determine whether R is an equivalence relation.
- e. Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$, $n \geq 3$ with $a_1 = 3$ & $a_2 = 27$.
- f. For the recurrence relation $a_n = a_{n-1} + a_{n-2}$, $n \geq 3$, $a_1 = a_2 = 1$. Find a_6 & a_7 also find the characteristic equation.

Q.2 Attempt **any four** of the following: **20**

- a. In the expansion of $(x_1 + 2x_2 + x_3)^4$ find what is the coefficient of the following terms?
 i) $x_1 x_2^2 x_3$ ii) $x_1 x_2 x_3^2$ iii) $x_1^2 x_2 x_3^2$
- b. A committee of 12 is to be selected from 10 men and 10 women. In how many ways the selection can be carried out if there must be six men and six women?
- c. State generalised pigeonhole principle and hence show that in any group of 6 people there are 3 mutual friends or 3 mutual strangers.
- d. Defined the cardinality of a set. Also, find the cardinal number of each set.
 i) $A = \{1, 3, 5, 7, 11, 13\}$ ii) $B = \{a, b, c, \dots \dots \dots, y, z\}$

iii) $C = \{x / x^2 = 4 \text{ or } x^2 = 7\}$

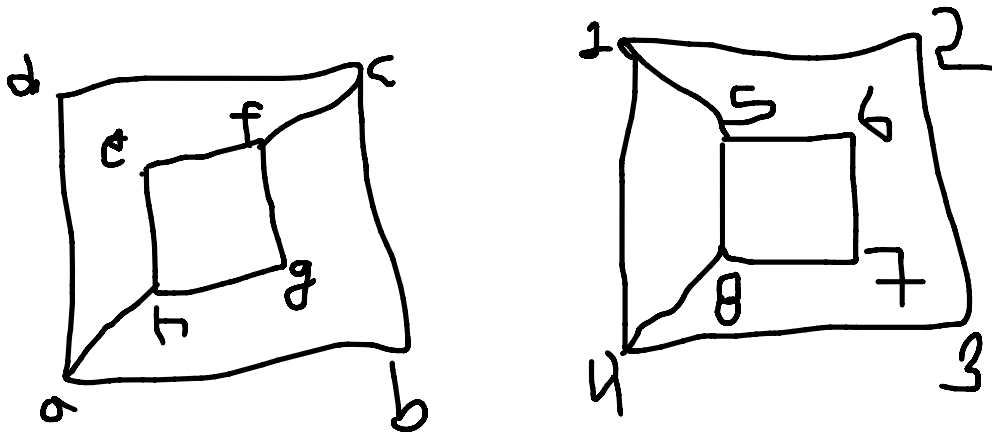
iv) $D = \{5, 10, 15, 20, 25, 30, \dots \dots \dots\}$

- e. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that contain the string 101.
- f. Construct a deterministic finite-state automaton that recognizes the set of all bit strings that begin with 0 or with 11.

Q.3 Attempt any four of the following:

20

- a. Define Complete and regular graph and explain the difference between them.
- b. Check whether the following graphs are isomorphic or not.



- c. Draw an undirected graph represented by the given adjacency matrix and find the degree of each of its vertex.

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

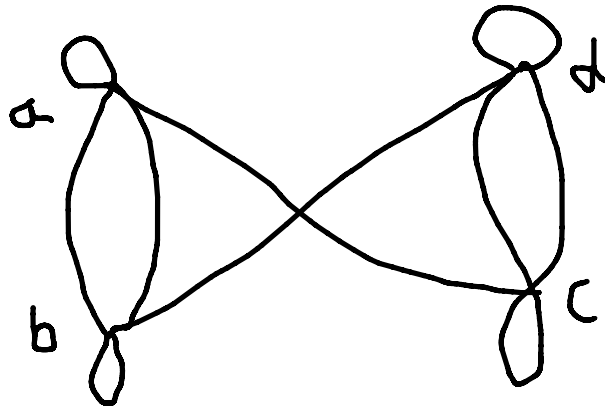
- d. If a Binary tree has 200 nodes, then find the number of leaf nodes.
- e. Build binary search tree for the number 17, 12, 15, 21, 26, 8, 20, 11, 30.
- f. Form a Binary search tree for the words in the sentence "The Quick brown fox jumps over the lazy dog." Using the alphabetical order.

Q.4 Attempt any three of the following:

15

- a. Let $X = \{0, 1, 2, 3, 4\}$ $Y = \{a, b, c, d\}$ and $f(0) = a = f(1)$, $f(2) = b$, $f(3) = c = f(4)$. Find

- a) $f(\{1\} \cap \{2\})$, b) $f^{-1}(\{a, c\})$.
- b. In \mathbb{N} $a * b = a^b$. Check whether $*$ is commutative, associative. Justify your answer.
- c. How many different letters word can be formed using the letters of word “MISSISSIPPI”?
- d. Find the number of integers from 1 to 150 which are not divisible by 2, 3 and 7.
- e. Represent the given graph using an adjacency matrix.



- f. Build a binary search tree for the words banana, peach, apple, pear, coconut, mango, and papaya using alphabetical order.
